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Proc. R. Soc. A 2011 **467**, 3033-3034 first published online 23 February 2011
doi: 10.1098/rspa.2011.0070

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Proc. R. Soc. A (2011) **467**, 3033–3034
doi:10.1098/rspa.2011.0070
Published online 23 February 2011

CORRECTION

Proc. R. Soc. A **462**, 2625–2641 (8 September 2006)
(doi:10.1098/rspa.2005.1633)

A multiplicity result for a nonlinear degenerate problem arising in the theory of electrorheological fluids

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The goal of this correction is to correct a mistake that appears in the proof of both lemma 3.9 and 3.10 in the article Mihăilescu & Rădulescu (2006).

The mistake in the proof of lemma 3.9 and 3.10 is mainly owing to the fact that we wrongly computed the expression of function $G(x,t)$ when $t > u_1(x)$. The correct computation reads as follows:

$$G(x,t) = \frac{1}{\gamma}u_1(x)^\gamma - \frac{1}{\beta}u_1(x)^\beta + (u_1(x)^{\gamma-1} - u_1(x)^{\beta-1})(t - u_1(x)), \quad \forall x \in \Omega,\ \ t > u_1(x). \tag{1.1}$$

We remember the result of Mihăilescu & Rădulescu (2006, lemma 3.9) and provide the correct proof.

Lemma 3.9. *There exists $\rho \in (0, \|u_1\|)$ and $a > 0$ such that $J(u) \geq a$, for all $u \in E$ with $\|u\| = \rho$.*

Proof. Let $u \in E$ be fixed, such that $\|u\| < 1$. It is clear that

$$\frac{1}{\gamma}t^\gamma - \frac{1}{\beta}t^\beta \leq 0, \quad \forall t \in [0,1].$$

Define

$$\Omega_u := \{x \in \Omega; u(x) > \min\{1, u_1(x)\}\}.$$

If $x \in \Omega \setminus \Omega_u$ then $u(x) \leq \min\{1, u_1(x)\} \leq u_1(x)$ and we have

$$G(x,u) = \frac{1}{\gamma}u_+^\gamma - \frac{1}{\beta}u_+^\beta \leq 0.$$

If $x \in \Omega_u \cap \{x; u_1(x) < u(x) < 1\}$ then

$$G(x,u) = \frac{1}{\gamma}u_1^\gamma - \frac{1}{\beta}u_1^\beta + (u_1^{\gamma-1} - u_1^{\beta-1})(u - u_1) \leq 0.$$

Define

$$\Omega_{u,1} := \Omega_u \setminus \{x; u_1(x) < u(x) < 1\}.$$

Thus, provided that $\|u\| < 1$ by condition (A5), the above estimates and relation (1.5) we get

$$\begin{aligned} J(u) &\geq \int_\Omega \frac{1}{p(x)} |\nabla u|^{p(x)}\,\mathrm{d}x - \lambda \int_{\Omega_{u,1}} G(x,u)\,\mathrm{d}x \\ &\geq \frac{1}{p^+}\|u\|^{p^+} - \lambda \int_{\Omega_{u,1}} G(x,u)\,\mathrm{d}x. \end{aligned} \tag{1.2}$$

Since $p^+ < \min\{N, Np^-/(N-p^-)\}$, it follows that $p^+ < p^\star(x)$ for all $x \in \boldsymbol{\Omega}$. Then, there exists $q \in (p^+, Np^-/(N-p^-))$ such that E is continuously embedded in $L^q(\Omega)$. Thus, there exists a positive constant $C > 0$ such that

$$|u|_q \leq C\|u\|, \quad \forall u \in E.$$

Using the definition of G and the above estimate, we obtain

$$\begin{aligned} \lambda \int_{\Omega_{u,1}} G(x,u)\,\mathrm{d}x &= \lambda \int_{\Omega_{u,1}\cap[u<u_1]} \left(\frac{1}{\gamma}u_+^\gamma - \frac{1}{\beta}u_+^\beta\right)\mathrm{d}x + \lambda \int_{\Omega_{u,1}\cap[u>u_1]} \left(\frac{1}{\gamma}u_1^\gamma - \frac{1}{\beta}u_1^\beta\right)\mathrm{d}x \\ &\quad + \lambda \int_{\Omega_{u,1}\cap[u>u_1]} (u_1^{\gamma-1} - u_1^{\beta-1})(u-u_1)\,\mathrm{d}x \leq \frac{\lambda}{\gamma} \int_{\Omega_{u,1}\cap[u<u_1]} \frac{1}{\gamma} u_+^\gamma\,\mathrm{d}x \\ &\quad + \frac{\lambda}{\gamma} \int_{\Omega_{u,1}\cap[u>u_1]} u_1^\gamma\,\mathrm{d}x + \lambda \int_{\Omega_{u,1}\cap[u>u_1]} u_1^{\gamma-1} u\,\mathrm{d}x \\ &\leq \lambda D \int_{\Omega_{u,1}} u_+^\gamma\,\mathrm{d}x \leq \lambda D \int_{\Omega_{u,1}} u_+^q\,\mathrm{d}x \leq \lambda D_1 \|u\|^q, \end{aligned} \tag{1.3}$$

where D and D_1 are two positive constants. Combining inequalities (1.2) and (1.3), we find that for a $\rho \in (0, \min\{1, \|u_1\|\})$ small enough we have

$$J(u) \geq \left(\frac{1}{p^+} - \lambda D_1 \|u\|^{q-p^+}\right)\|u\|^{p^+},$$

and taking into account that $q > p^+$, we infer that the conclusion of this lemma holds true. ■

Regarding the proof of Mihăilescu & Rădulescu (2006, lemma 3.10) it can be carried out with the same arguments as in the original proof, but the expression of $G(x,t)$ with $t > u_1(x)$ should be replaced in the proof with the one given by relation (1.1).

References

Mihăilescu, M. & Rădulescu, V. 2006 A multiplicity result for a nonlinear degenerate problem arising in the theory of electrorheological fluids. *Proc. R. Soc. A* **462**, 2625–2641. (doi:10.1098/rspa.2005.1633)