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POSITIVE BOUNDED SOLUTIONS FOR SEMILINEAR ELLIPTIC SYSTEMS WITH INDEFINITE WEIGHTS IN THE HALF-SPACE

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Abstract. In this article, we study the existence and nonexistence of positive bounded solutions of the Dirichlet problem

$$\begin{gathered} -\Delta u = \lambda p(x) f(u,v), \quad \text{in } \mathbb{R}^n_+, \\ -\Delta v = \lambda q(x) g(u,v), \quad \text{in } \mathbb{R}^n_+, \\ u = v = 0 \quad \text{on } \partial\mathbb{R}^n_+, \\ \lim_{|x|\to\infty} u(x) = \lim_{|x|\to\infty} v(x) = 0, \end{gathered}$$

where $\mathbb{R}^n_+ = \{x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_n > 0\}$ $(n \geq 3)$ is the upper half-space and λ is a positive parameter. The potential functions p, q are not necessarily bounded, they may change sign and the functions $f, g : \mathbb{R}^2 \to \mathbb{R}$ are continuous. By applying the Leray-Schauder fixed point theorem, we establish the existence of positive solutions for λ sufficiently small when $f(0,0) > 0$ and $g(0,0) > 0$. Some nonexistence results of positive bounded solutions are also given either if λ is sufficiently small or if λ is large enough.

1. Introduction

This paper deals with the existence of positive continuous solutions (in the sense of distributions) for the semilinear elliptic system

$$\begin{gathered} -\Delta u = \lambda p(x) f(u,v), \quad \text{in } \mathbb{R}^n_+, \\ -\Delta v = \lambda q(x) g(u,v), \quad \text{in } \mathbb{R}^n_+, \\ u = v = 0 \quad \text{on } \partial\mathbb{R}^n_+, \\ \lim_{|x|\to\infty} u(x) = \lim_{|x|\to\infty} v(x) = 0, \end{gathered} \tag{1.1}$$

where $\mathbb{R}^n_+ = \{x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_n > 0\}$ $(n \geq 3)$ is the upper half-space. We assume that the potentials p, q are sign-changing functions belonging to the Kato class $K^\infty(\mathbb{R}^n_+)$ introduced and studied in [1], and the functions f, g satisfy the following hypothesis:

(H1) $f, g : \mathbb{R}^2 \to \mathbb{R}$ are continuous with $f(0,0) > 0$ and $g(0,0) > 0$.

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In recent years, a good amount of research is established for reaction-diffusion systems. Reaction-diffusions systems model many phenomena in biology, ecology, combustion theory, chemical reactors, population dynamics etc. The case $p(x) = q(x) = 1$ has been considered as a typical example in bounded regular domains in $\mathbb{R}^n$ and many existence results where established by variational methods, topological methods and the method of sub- and super-solutions (see [4, 7, 5, 6, 8]).

Recently, Chen [2] studied the existence of positive solutions for the system

$$\begin{gathered} -\Delta u = \lambda p(x) f_1(v), \quad \text{in } D, \\ -\Delta v = \lambda q(x) g_1(v), \quad \text{in } D, \\ u = v = 0 \quad \text{on } \partial D, \end{gathered} \tag{1.2}$$

where D is a bounded domain. He assumed that p, q are continuous in $\overline{D}$ and there exist positive constants μ_1, μ_2 such that

$$\begin{gathered} \int_D G_D(x,y) p_+(y)\,dy > (1+\mu_1)\int_D G_D(x,y) p_-(y)\,dy \quad \forall x \in D, \\ \int_D G_D(x,y) q_+(y)\,dy > (1+\mu_2)\int_D G_D(x,y) q_-(y)\,dy \quad \forall x \in D, \end{gathered}$$

where $G_D(x,y)$ is the Green's function of the Dirichlet Laplacian in D. Here p^+, q^+ are the positive parts of p and q, while p_-, q_- are the negative ones. Chen [2] showed that if $f_1, g_1 : [0,\infty) \to \mathbb{R}$ are continuous with $f_1(0) > 0$, $g_1(0) > 0$ and p, q are nonzero continuous functions on $\overline{D}$ satisfying the above integral conditions, then there exists a positive number $\lambda^\star$ such that problem (1.2) has a positive solution for small positive values of the parameter, namely if $0 < \lambda < \lambda^\star$.

We note that when f_1, g_1 are nonnegative nondecreasing continuous functions, $p(x) \le 0$ in $\mathbb{R}^n_+$ and $q(x) \le 0$ in $\mathbb{R}^n_+$, system (1.2) was studied in [10] in the half-space $\mathbb{R}^n_+$ with nontrivial nonnegative boundary and infinity data. In this framework, the existence of positive solutions for (1.2) is established for small perturbations, that is, whenever λ is a small positive real number.

Our aim in this article is to study these systems in the case where the domain is the half-space $\mathbb{R}^n_+$ and the functions p, q are not necessarily continuous in $\overline{\mathbb{R}^n_+}$. Indeed p, q may be singular on the boundary of $\mathbb{R}^n_+$. More precisely, we establish the existence of a positive bounded solution for (1.1) in the case where $f(0,0) > 0$, $g(0,0) > 0$ and the functions p, q belong to the Kato class introduced and studied in [1] and satisfy the following hypothesis:

(H2) there exist positive numbers μ_1, μ_2 such that

$$\begin{gathered} \int_{\mathbb{R}^n_+} G(x,y) p_+(y)\,dy > (1+\mu_1)\int_{\mathbb{R}^n_+} G(x,y) p_-(y)\,dy \quad \forall x \in \mathbb{R}^n_+, \\ \int_{\mathbb{R}^n_+} G(x,y) q_+(y)\,dy > (1+\mu_2)\int_{\mathbb{R}^n_+} G(x,y) q_-(y)\,dy \quad \forall x \in \mathbb{R}^n_+, \end{gathered}$$

where $G(x,y)$ is the Green function of the Dirichlet Laplacian in the half space $\mathbb{R}^n_+$.

Two nonexistence results of positive bounded solutions will be established in this paper. To this aim, we recall in the sequel some notations and properties of the Kato class, cf. [1].

Definition 1.1. A Borel measurable function k in $\mathbb{R}^n_+$ belongs to the Kato class $K^\infty(\mathbb{R}^n_+)$ if

$$\lim_{r\to 0}\sup_{x\in\mathbb{R}^n_+}\int_{\mathbb{R}^n_+\cap B(x,r)}\frac{y_n}{x_n}G(x,y)|k(y)|dy = 0$$

and

$$\lim_{M\to\infty}\sup_{x\in\mathbb{R}^n_+}\int_{\mathbb{R}^n_+\cap\{|y|\geq M\}}\frac{y_n}{x_n}G(x,y)|k(y)|dy = 0\,,$$

where

$$G(x,y)=\frac{\Gamma(\frac{n}{2}-1)}{4\pi^{n/2}}\Big[\frac{1}{|x-y|^{n-2}}-\frac{1}{(|x-y|^2+4x_ny_n)^{\frac{n-2}{2}}}\Big]$$

is the Green function of the Dirichlet Laplacian in $\mathbb{R}^n_+$.

Next, we give some examples of functions belonging to $K^\infty(\mathbb{R}^n_+)$.

Example 1.2. Let $\lambda,\mu\in\mathbb{R}$ and put $q(y)=\frac{1}{(|y|+1)^{\mu-\lambda}y_n^\lambda}$ for $y\in\mathbb{R}^n_+$. Then

$$q\in K^\infty(\mathbb{R}^n_+) \text{ if and only if } \lambda<2<\mu.$$

For any nonnegative Borel measurable function φ in $\mathbb{R}^n_+$, we denote by $V\varphi$ the Green potential of φ:

$$V\varphi(x)=\int_{\mathbb{R}^n_+}G(x,y)\varphi(y)dy,\quad \forall x\in\mathbb{R}^n_+.$$

Recall that if $\varphi\in L^1_{\rm loc}(\mathbb{R}^n_+)$ and $V\varphi\in L^1_{\rm loc}(\mathbb{R}^n_+)$, then we have in the distributional sense (see [3, p. 52])

$$\Delta(V\varphi)=-\varphi \text{ in } \mathbb{R}^n_+. \tag{1.3}$$

The first result establishes the existence of bounded positive solutions in case of small perturbations, that is, if λ is a small positive parameter.

Theorem 1.3. *Let p, q be in the Kato class $K^\infty(\mathbb{R}^n_+)$ and assume that* (H1)–(H2) *are satisfied. Then there exists $\lambda_0>0$ such that for each $\lambda\in(0,\lambda_0)$, problem* (1.1) *has a positive continuous solution in $\mathbb{R}^n_+$.*

The first nonexistence result of positive bounded solutions is in relationship with the previous theorem and concerns a particular class of functions f and g with linear growth and vanishing at the origin.

Theorem 1.4. *Let p,q be nontrivial functions in the Kato class $K^\infty(\mathbb{R}^n_+)$. Assume that the functions f, $g:\mathbb{R}^2\to\mathbb{R}$ are measurable and there exists a positive constant M such that for all u, v*

$$|f(u,v)|\leq M(|u|+|v|)$$
$$|g(u,v)|\leq M(|u|+|v|).$$

Then there exists $\lambda_0>0$ such that problem (1.1) *has no positive bounded continuous solution in $\mathbb{R}^n_+$ for each $\lambda\in(0,\lambda_0)$.*

The second nonexistence result is established for λ sufficiently large.

Theorem 1.5. *Let $p,q\in K^\infty(\mathbb{R}^n_+)$ and let $f(u,v)=f(v)$, $g(u,v)=g(u)$. Assume that the following hypotheses are fulfilled:*

(H3) *there exist an open ball $B\subset\mathbb{R}^n_+$ and a positive number ε such that*

$$p(x),q(x)\geq\varepsilon \quad \text{a.e. } x\in B.$$

(H4) *$f, g : [0, \infty) \to [0, \infty)$ are continuous and there exists a positive number m such that $f(v) + g(u) \geq m(u + v)$ for all $u, v > 0$.*

Then there exists a positive number λ_0 such that problem (1.1) *has no positive bounded continuous solution in $\mathbb{R}^n_+$ for each $\lambda > \lambda_0$.*

Throughout this article, we denote by $B(\mathbb{R}^n_+)$ the set of Borel measurable functions in $\mathbb{R}^n_+$ and by $C_0(\mathbb{R}^n_+)$ the set of continuous functions satisfying

$$\lim_{x \to \partial \mathbb{R}^n_+} u(x) = \lim_{|x| \to \infty} u(x) = 0.$$

Finally, for a bounded real function ω defined on a set S we denote $\|\omega\|_\infty = \sup_{x \in S} |\omega(x)|$.

2. Proof of main results

We start this section with the following continuity property. We refer to [1] for more details.

Proposition 2.1. *Let φ be a nonnegative function in $K^\infty(\mathbb{R}^n_+)$. Then the following properties hold.*

(i) *The function $y \to \frac{y_n}{(1+|y|)^n} \varphi(y)$ is in $L^1(\mathbb{R}^n_+)$, hence $\varphi \in L^1_{\rm loc}(\mathbb{R}^n_+)$.*
(ii) *$V\varphi \in C_0(\mathbb{R}^n_+)$.*
(iii) *Let h_0 be a positive harmonic function in $\mathbb{R}^n_+$ which is continuous and bounded in $\overline{\mathbb{R}^n_+}$. Then the family of functions*

$$\Big\{ \int_{\mathbb{R}^n_+} G(., y) h_0(y) p(y) dy : |p| \leq \varphi \Big\}$$

is relatively compact in $C_0(\mathbb{R}^n_+)$.

Next, we recall the Leray-Schauder fixed point theorem.

Lemma 2.2. *Let X be a Banach space with norm $\|\cdot\|$ and x_0 be a point of X. Suppose that $T : X \times [0, 1] \to X$ is continuous and compact with $T(x, 0) = x_0$ for each $x \in X$, and that there exists a fixed constant $M > 0$ such that each solution $(x, \sigma) \in X \times [0, 1]$ of the $T(x, \sigma) = x$ satisfies $\|x\| \leq M$. Then $T(., 1)$ has a fixed point.*

Using this fixed point property, we obtain the following general existence result.

Lemma 2.3. *Suppose that p and q are in the Kato class $K(\mathbb{R}^n_+)$ and f, g are continuous and bounded from $\mathbb{R}^2$ to $\mathbb{R}$. Then for every $\lambda \in (0, \infty)$, problem* (1.1) *has a solution $(u_\lambda, v_\lambda) \in C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+)$.*

Proof. For $\lambda \in \mathbb{R}$, we consider the operator

$$T_\lambda : C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+) \times [0, 1] \to C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+)$$

defined by

$$T_\lambda((u, v), \sigma) = (\sigma\lambda V(pf(u, v)), \sigma\lambda V(qg(u, v))).$$

By Proposition 2.1, the operator T_λ is well defined, continuous, compact and

$$T_\lambda((u, v), 0) = (0, 0) := x_0 \in C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+).$$

Let $(u,v) \in C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+)$ and $\sigma \in [0,1]$ such that $T_\lambda((u,v),\sigma) = (u,v)$. Then, since f,g are bounded and p, q are in $K^\infty(\mathbb{R}^n_+)$ we deduce by using Proposition 2.1 that

$$\begin{aligned}\max(\|u\|_\infty, \|v\|_\infty) &= \sigma\,\lambda \max(\|V(pf(u,v))\|_\infty, \|V(qg(u,v))\|_\infty) \\ &\le \lambda \max(\|Vp\|_\infty \|f\|_\infty, \|Vq\|_\infty \|g\|_\infty) = M.\end{aligned}$$

Applying the Leray-Schauder fixed point theorem, the operator $T_\lambda(.,1)$ has a fixed point, hence there exists $(u,v) \in C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+)$ such that

$$(u,v) = (\lambda V(p\,f(u,v)), \lambda V(q\,g(u,v))).$$

So, using (1.3) and Proposition 2.1, we deduce that (u,v) is a solution of system (1.1). □

Proof of Theorem 1.3. Fix a large number $M > 0$ and an infinitely continuously differentiable function ψ with compact support on $\mathbb{R}^2$ such that $\psi = 1$ in the open ball with center 0 and radius M and $\psi = 0$ on the exterior of the ball with center 0 and radius $2M$.

Define the bounded functions $\widetilde{f}$, $\widetilde{g}$ on $\mathbb{R}^2$ by

$$\widetilde{f}(u,v) = \psi(u,v) f(u,v) \quad \text{and} \quad \widetilde{g}(u,v) = \psi(u,v) g(u,v).$$

By Lemma 2.3, the Dirichlet problem

$$\begin{gathered} -\Delta u = \lambda p(x) \widetilde{f}(u,v), \quad \text{in } \mathbb{R}^n_+, \\ -\Delta v = \lambda q(x) \widetilde{g}(u,v), \quad \text{in } \mathbb{R}^n_+, \\ u = v = 0 \quad \text{on } \partial\mathbb{R}^n_+, \\ \lim_{|x|\to\infty} u(x) = \lim_{|x|\to\infty} v(x) = 0, \end{gathered} \tag{2.1}$$

has a solution $(u_\lambda, v_\lambda) \in C_0(\mathbb{R}^n_+) \times C_0(\mathbb{R}^n_+)$ satisfying

$$(u_\lambda, v_\lambda) = (\lambda V(p\widetilde{f}(u_\lambda, v_\lambda)) \lambda V(q\widetilde{g}(u_\lambda, v_\lambda))).$$

Moreover, we have

$$\max(\|u_\lambda\|_\infty, \|v_\lambda\|_\infty) \le \lambda \max(\|Vp\|_\infty \|\widetilde{f}\|_\infty, \|Vq\|_\infty \|\widetilde{g}\|_\infty), \tag{2.2}$$

Put $\mu = \min(\mu_1, \mu_2)$ and consider $\gamma \in (0, \frac{\mu}{2+\mu})$. Since $\widetilde{f}$ and $\widetilde{g}$ are continuous, then there exists $\delta \in (0, M)$ such that if $\max(|\zeta|, |\xi|) < \delta$, we have

$$\begin{gathered} \widetilde{f}(0,0)(1-\gamma) < \widetilde{f}(\zeta,\xi) < \widetilde{f}(0,0)(1+\gamma), \\ \widetilde{g}(0,0)(1-\gamma) < \widetilde{g}(\zeta,\xi) < \widetilde{g}(0,0)(1+\gamma). \end{gathered}$$

Using relation (2.2), we deduce that there exists $\lambda_0 > 0$ such that $\|u_\lambda\|_\infty < \delta$ and $\|v_\lambda\|_\infty < \delta$ for any $\lambda \in (0, \lambda_0)$. This together with the fact that $0 < \delta < M$ implies that for $\lambda \in (0, \lambda_0)$, we have $\widetilde{f}(u_\lambda, v_\lambda) = f(u_\lambda, v_\lambda)$ and $\widetilde{g}(u_\lambda, v_\lambda) = g(u_\lambda, v_\lambda)$. Now, for each $x \in D$ we have

$$\begin{aligned} u_\lambda &= \lambda V(p_+ \widetilde{f}(u_\lambda, v_\lambda)) - \lambda V(p_- \widetilde{f}(u_\lambda, v_\lambda)) \\ &> \lambda f(0,0)(1-\gamma) V(p_+) - \lambda f(0,0)(1+\gamma) V(p_-) \\ &> \lambda f(0,0)[(1-\gamma)(1+\mu_1) - (1+\gamma)] V(p_-) \\ &> \lambda f(0,0)(1-\gamma)\big[1 + \mu_1 - \frac{1+\gamma}{1-\gamma}\big] V(p_-) \end{aligned}$$

$$> \lambda f(0,0)(1-\gamma)\big[1+\mu-\frac{1+\gamma}{1-\gamma}\big]V(p_-).$$

Now, since $\gamma \in (0, \frac{\mu}{2+\mu})$, then $1+\mu-\frac{1+\gamma}{1-\gamma} > 0$ and it follows that

$$\lambda f(0,0)(1-\gamma)\big[1+\mu-\frac{1+\gamma}{1-\gamma}\big]V(p_-) \geq 0.$$

Consequently, for each $\lambda \in (0,\lambda_0)$ and for each $x \in \mathbb{R}^n_+$ we have $u_\lambda(x) > 0$. Similarly, we obtain $v_\lambda(x) > 0$ for each $x \in \mathbb{R}^n_+$. □

Proof of Theorem 1.4. Suppose that problem (1.1) has a bounded positive solution (u,v) for all $\lambda > 0$. Then $f(u,v)$ and $g(u,v)$ are bounded. Put $\widetilde{u} = \lambda V(p\, f(u,v))$ and $\widetilde{v} = \lambda V(q\, g(u,v))$. Since $f(u,v)$ and $g(u,v)$ are bounded, it follows that $\widetilde{u}\,,\, \widetilde{v} \in C_0(\mathbb{R}^n_+)$. The the functions $z = u-\widetilde{u}$ and $\omega = v-\widetilde{v}$ are harmonic in the distributional sense and continuous in $\mathbb{R}^n_+$, so they are harmonic in the classical sense. Moreover, since $u = \widetilde{u} = v = \widetilde{v} = 0$ on $\partial\mathbb{R}^n_+$ and $\lim_{|x|\to\infty} u(x) = \lim_{|x|\to\infty} v(x) = 0$, then $u = \widetilde{u}$ and $v = \widetilde{v}$ in $\mathbb{R}^n_+$. It follows that

$$\|u\|_\infty \leq \lambda\, V(|p| f(u,v)) \leq \lambda M \|V(|p|)\|_\infty \left(\|u\|_\infty + \|v\|_\infty\right),$$
$$\|v\|_\infty \leq \lambda\, V(|q| g(u,v)) \leq \lambda M \|V(|q|)\|_\infty \left(\|u\|_\infty + \|v\|_\infty\right).$$

By adding these inequalities, we obtain

$$(\|u\|_\infty + \|v\|_\infty) \leq \lambda M \left[\|V(|p|)\|_\infty + \|V(|q|)\|_\infty\right] (\|u\|_\infty + \|v\|_\infty).$$

This gives a contradiction if $\lambda M[\|V(|p|)\|_\infty + \|V(|q|)\|_\infty] < 1$. □

Proof of Theorem 1.5. Without loss of generality, we assume that $\overline{B} \subset \Omega$. We first note that the assumption (H4) implies that

$$f(v) \geq mv \quad \text{for all } v > 0$$

or

$$g(u) \geq mu \quad \text{for all } u > 0.$$

Suppose that $f(v) \geq mv$ for all $v > 0$. We distinguish the following situations.

Case 1. $f(0) = 0$. Then it follows from (H4) that

$$g(u) \geq mu \quad \text{for } u > 0.$$

Suppose that (u,v) is a positive solution of (1.1). It follows that

$$-\Delta u = \lambda a(x) f(v) \geq \lambda\varepsilon m v \quad \text{in } B. \tag{2.3}$$

Let $\widetilde{\lambda}_1$ be the first eigenvalue of $-\Delta$ in B with Dirichlet boundary conditions, and ϕ_1 be the corresponding normalized positive eigenfunction. Let $\delta > 0$ be the largest number so that

$$v \geq \delta\phi_1 \quad \text{in } B. \tag{2.4}$$

Then we have from (2.3) and (2.4) that

$$-\Delta v \geq \lambda\varepsilon m\delta\phi_1 \quad \text{in } B,$$

and therefore by the weak comparison principle

$$u \geq \frac{\lambda\varepsilon m}{\widetilde{\lambda}_1}\delta\phi_1 \quad \text{in } B. \tag{2.5}$$

Therefore,

$$-\Delta v \geq \lambda\varepsilon m u \geq \frac{(\lambda\varepsilon m)^2}{\widetilde{\lambda}_1}\delta\phi_1 \quad \text{in } B.$$

Using by the weak comparison principle we obtain

$$v \geq \Big(\frac{\lambda\varepsilon m}{\widetilde{\lambda}_1}\Big)^2 \delta\phi_1 \quad \text{in } B.$$

This contradicts the maximality of δ for λ large enough.

Case 2. $f(0) > 0$. Then there exists $\delta_0 > 0$ such that

$$f(t) \geq \delta_0 \quad \text{for all } t \geq 0.$$

Hence $-\Delta u \geq \lambda\varepsilon\delta_0$ in B, from which it follows that

$$u \geq (\lambda\varepsilon\delta_0)\widetilde{\Phi} \quad \text{in } B, \tag{2.6}$$

where $\widetilde{\Phi}$ satisfies

$$-\Delta\widetilde{\Phi} = 1 \quad \text{in } B, \quad \widetilde{\Phi} = 0 \quad \text{on } \partial B.$$

Let D be an open set such that $\overline{D} \subset B$ and let $c > 0$ such that

$$\widetilde{\Phi} \geq c \quad \text{in } \overline{D}. \tag{2.7}$$

Suppose $m\lambda\varepsilon\delta_0 c > 2f(0)$. Relations (2.6) and (2.7) yield

$$mu \geq m\lambda\varepsilon\delta_0 c > 2f(0),$$

which implies

$$g(u) \geq mu - f(0) \geq \frac{m}{2}u \quad \text{in } D.$$

Using the same arguments as in Case 1 in D, we obtain a contradiction if λ is large enough. The case when $g(u) \geq mu$ for all $u > 0$ is treated in a similar manner. This completes the proof. □

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