Houston Journal of Mathematics

Volume 29, No. 3, 2003

SOLUTIONS WITH BOUNDARY BLOW-UP FOR A CLASS OF NONLINEAR ELLIPTIC PROBLEMS

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Communicated by Haïm Brezis

Abstract. Let Ω be a smooth bounded domain in $\mathbf{R}^N$. We consider the logistic equation $\Delta u + au = b(x)f(u)$ in Ω, where a is a real number, b is continuous, $b \geq 0$, $b \not\equiv 0$, and $f \in C^1$ is a positive function satisfying the Keller–Osserman condition and such that $f(u)/u$ is increasing on $(0, \infty)$. We prove that a necessary and sufficient condition for the existence of a positive solution blowing-up at the boundary of Ω is that $a \in (-\infty, \lambda_{\infty,1})$, where $\lambda_{\infty,1}$ is the first eigenvalue of $(-\Delta)$ in $H_0^1(\Omega_0)$ and $\Omega_0 = \text{int}\,\{x \in \Omega;\ b(x) = 0\}$. Our framework includes the case when the potential b vanishes at some points on $\partial\Omega$ or even on the whole boundary.

1. The Main Result

This paper originated with the recent work Alama–Tarantello [1] which contains an exhaustive study of the logistic problem

$$(1)\qquad \begin{cases} \Delta u + \lambda u = b(x)f(u) & \text{in } \Omega\,, \\ u = 0 & \text{on } \partial\Omega\,, \\ u > 0 & \text{in } \Omega\,, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbf{R}^N$ $(N \geq 2)$, λ is a real parameter and $b \in C^{0,\alpha}(\overline{\Omega})$, $0 < \alpha < 1$ satisfies $b \geq 0$ and $b \not\equiv 0$ in Ω. It is worth pointing out here that if $f(u) = u^{(N+2)/(N-2)}$ (for $N \geq 3$), then this equation originates from

2000 *Mathematics Subject Classification.* 35J60; 35B40; 35B50.

Key words and phrases. Explosive solution, logistic equation, existence result, maximum principle.

The research of F. Cîrstea was done under the IPRS Programme funded by the Australian Government through DETYA. V. Rădulescu was supported by the P.I.C.S. Research Programme between France and Romania.

the Yamabe problem, which is a basic problem in Riemannian geometry (see, e.g., [9]).

The zero set

$$\Omega_0 := \text{int}\,\{x \in \Omega : \ b(x) = 0\}$$

plays an important role in the understanding of this problem. We shall assume throughout that $\overline{\Omega}_0 \subset \Omega$ and $b > 0$ in $\Omega \setminus \overline{\Omega}_0$.

Suppose that $f \in C^1[0,\infty)$ satisfies

(f_1) $\quad f \geq 0$ and $f(u)/u$ is increasing on $(0,\infty)$.

Following Alama–Tarantello [1], define by H_∞ the Dirichlet Laplacian on the set $\Omega_0 \subset \Omega$ as the unique self-adjoint operator associated to the quadratic form $\xi(u) = \int_\Omega |\nabla u|^2\,dx$ with form domain

$$H^1_D(\Omega_0) = \{u \in H^1_0(\Omega) : \ u(x) = 0 \quad \text{for a.e. } x \in \Omega \setminus \Omega_0\}.$$

If $\partial\Omega_0$ satisfies an exterior cone condition, then $H^1_D(\Omega_0)$ coincides with $H^1_0(\Omega_0)$ and H_∞ is the classical Laplace operator with Dirichlet condition on $\partial\Omega_0$ (see [1]).

Let $\lambda_{\infty,1}$ be the first Dirichlet eigenvalue of H_∞ in Ω_0. We understand $\lambda_{\infty,1} = +\infty$ if $\Omega_0 = \emptyset$.

Set $\mu_0 := \lim_{u\searrow 0} \frac{f(u)}{u}$ and $\mu_\infty := \lim_{u\to\infty} \frac{f(u)}{u}$. The results of Alama and Tarantello rely on the existence of a principal eigenvalue for the operator $-\Delta + \mu b$ in the limiting cases $\mu = \mu_0$ and $\mu = \mu_\infty$. Denote by $\lambda_1(\mu_0)$ (resp., $\lambda_1(\mu_\infty)$) the first eigenvalue of $H_{\mu_0} = -\Delta + \mu_0 b$ (resp., $H_{\mu_\infty} = -\Delta + \mu_\infty b$) in $H^1_0(\Omega)$. Recall that $\lambda_1(+\infty) = \lambda_{\infty,1}$.

The main result of [1] (see also [6], [16]) asserts that problem (1) has a solution u_λ if and only if $\lambda \in (\lambda_1(\mu_0), \lambda_1(\mu_\infty))$, and, moreover, u_λ is the unique solution of (1) (see [1, Theorem A (bis)]). We point out that neither assumption on the smoothness of $\partial\Omega_0$ nor topological restriction on Ω are made in [1].

Our purpose is to give a corresponding necessary and sufficient condition, but for solutions of the problem

$$(2) \qquad \begin{cases} \Delta u + au = b(x)f(u) & \text{in } \Omega\,, \\ \lim\limits_{\text{dist}\,(x,\partial\Omega)\to 0} u(x) = +\infty\,, & \\ u \geq 0 & \text{in } \Omega\,, \end{cases}$$

where a is a real parameter. A solution of (2) is called *large* (or *explosive*) solution. There is a vast literature on nonlinear elliptic problems having solutions that blow-up at the boundary, starting with the pioneering papers [14], [8], [13], [10]. We also refer to the paper [15], where there are studied large solutions of the problem

$$\Delta u = b(x)u^{(N+2)/(N-2)}$$

in a ball, in particular for questions of existence, uniqueness and boundary behaviour.

We impose the natural Keller–Osserman condition

$$(f_2) \qquad \int_1^\infty \frac{dt}{\sqrt{F(t)}} < \infty\,, \quad \text{where } F(t) = \int_0^t f(s)\,ds.$$

We recall (see [8, 13]) that this condition is necessary and sufficient for the existence of a large solution to problem $\Delta u = h(u)$, where $h \in C^1$, $h(0) = 0$, $h' \geq 0$ and $h > 0$ on $(0, \infty)$.

Examples of non-linearities satisfying (f_1) and (f_2): (i) $f(u) = e^u - 1$; (ii) $f(u) = u^p$, $p > 1$; (iii) $f(u) = u\,[\ln(u+1)]^p$, $p > 2$.

REMARK 1. *We have* $\mu_\infty := \lim_{u\to\infty} f(u)/u = \lim_{u\to\infty} f'(u) = \infty$. Indeed, by l'Hospital's rule, we have $\lim_{u\to\infty} F(u)/u^2 = \mu_\infty/2$. But, by (f_2), we deduce that $\mu_\infty = \infty$. Then, by (f_1) we find that $f'(u) \geq f(u)/u$ for any $u > 0$, which shows that $\lim_{u\to\infty} f'(u) = \infty$.

Our main result is

Theorem 1.1. *Assume conditions (f_1) and (f_2) hold. Then problem (2) has a solution if and only if $a \in (-\infty, \lambda_{\infty,1})$. Moreover, in this case, the solution is positive.*

We point out that our framework in the above result includes the case when b vanishes at some points on $\partial\Omega$, or even if $b \equiv 0$ on $\partial\Omega$. In this sense, our result responds to a question raised to one of us by Professor Haim Brezis in Paris, May 2001.

Our result also applies to problems on Riemannian manifolds if Δ is replaced by the Laplace–Beltrami operator

$$\Delta_B = \frac{1}{\sqrt{c}}\,\frac{\partial}{\partial x_i}\left(\sqrt{c}\,a_{ij}(x)\frac{\partial}{\partial x_i}\right)\,, \qquad c := \det\left(a_{ij}\right),$$

with respect to the metric $ds^2 = c_{ij}\,dx_i dx_j$, where (c_{ij}) is the inverse of (a_{ij}). In this case our result applies to concrete problems arising in Riemannian geometry. For instance, (cf. Loewner-Nirenberg [10]) if Ω is replaced by the standard N–sphere (S^N, g_0), Δ is the Laplace-Beltrami operator Δ_{g_0}, $a = N(N-2)/4$, and $f(u) = (N-2)/[4(N-1)]\,u^{(N+2)/(N-2)}$, we find the prescribing scalar curvature equation on S^N.

2. An auxiliary comparison principle

Lemma 1. *Let $\omega \subset \mathbf{R}^N$ be a smooth bounded domain. Assume f is continuous on $(0,\infty)$, $f(u)/u$ is increasing on $(0,\infty)$, and p, q, r are $C^{0,\mu}$-functions on $\overline{\omega}$ such that $r \geq 0$ and $p > 0$ in ω. Let u_1, $u_2 \in C^2(\omega)$ be positive functions such that*

$$\Delta u_1+q(x)u_1-p(x)f(u_1)+r(x) \leq 0 \leq \Delta u_2+q(x)u_2-p(x)f(u_2)+r(x) \quad \text{in } \omega \tag{3}$$

$$\limsup_{\operatorname{dist}(x,\partial\omega)\to 0} (u_2-u_1)(x) \leq 0. \tag{4}$$

Then $u_1 \geq u_2$ in ω.

Proof. We use the same method as in the proof of Lemma 1.1 in Marcus-Véron [12] (see also [7, Lemma 2.1]), that goes back to Benguria-Brezis-Lieb [2].

By (3) we obtain, for any non-negative function $\phi \in H^1(\omega)$ with compact support in ω,

$$\int_\omega (\nabla u_1\cdot\nabla\phi - qu_1\phi + pf(u_1)\phi - r\phi) \geq 0 \geq \int_\omega (\nabla u_2\cdot\nabla\phi - qu_2\phi + pf(u_2)\phi - r\phi)\,. \tag{5}$$

Let $\varepsilon_1 > \varepsilon_2 > 0$ and denote

$$\omega(\varepsilon_1,\varepsilon_2) = \{x\in\omega:\ u_2(x)+\varepsilon_2 > u_1(x)+\varepsilon_1\}$$

$$v_i = (u_i+\varepsilon_i)^{-1}\left((u_2+\varepsilon_2)^2-(u_1+\varepsilon_1)^2\right)^+, \quad i=1,2.$$

Notice that $v_i \in H^1(\omega)$ and, in view of (4), it has compact support in ω. Using (5) with $\phi = v_i$ and taking into account that v_i vanishes outside $\omega(\varepsilon_1,\varepsilon_2)$ we find

$$\begin{aligned} -\int_{\omega(\varepsilon_1,\,\varepsilon_2)} (\nabla u_2\cdot\nabla v_2 - \nabla u_1\cdot\nabla v_1)\,dx &\geq \int_{\omega(\varepsilon_1,\,\varepsilon_2)} p(f(u_2)v_2 - f(u_1)v_1)\,dx \\ &+ \int_{\omega(\varepsilon_1,\,\varepsilon_2)} q(u_1v_1-u_2v_2)\,dx + \int_{\omega(\varepsilon_1,\,\varepsilon_2)} r(v_1-v_2)\,dx\,. \end{aligned} \tag{6}$$

A simple computation shows that the integral in the left-hand side of (6) equals

$$-\int_{\omega(\varepsilon_1,\,\varepsilon_2)}\left(\left|\nabla u_2 - \frac{u_2+\varepsilon_2}{u_1+\varepsilon_1}\nabla u_1\right|^2 + \left|\nabla u_1 - \frac{u_1+\varepsilon_1}{u_2+\varepsilon_2}\nabla u_2\right|^2\right)dx \leq 0\,.$$

Passing to the limit as $0<\varepsilon_2<\varepsilon_1\to 0$, the first term in the right hand-side converges to

$$\int_{\omega(0,\,0)} p\left(\frac{f(u_2)}{u_2}-\frac{f(u_1)}{u_1}\right)(u_2^2-u_1^2)\,dx\,,$$

the second term goes to 0, while the third one converges to

$$\int_{\omega(0,\,0)} \frac{r(u_2-u_1)^2(u_2+u_1)}{u_1u_2}\,dx \geq 0\,.$$

Hence we avoid a contradiction only in the case when $\omega(0,0)$ has measure 0, which means that $u_1 \geq u_2$ on ω. □

3. Proof of Theorem 1.1

A. Necessary condition. Let u_∞ be a large solution of problem (2). We claim that u_∞ is positive. Indeed, since $u_\infty(x) \to \infty$ as $\operatorname{dist}(x, \partial\Omega) \to 0$, there exists a smooth open set $\omega \subset\subset \Omega$ such that $u_\infty > 0$ on $\Omega\backslash\omega$. So, it is enough to show that $u_\infty > 0$ in $\overline{\omega}$. For this aim, set $M_0 := 1 + \sup_\omega b > 0$ and consider the problem

$$\begin{cases} \Delta u = |a|u + M_0 f(u) & \text{in } \omega\,,\\ u = u_\infty & \text{on } \partial\omega\,,\\ u \geq 0 & \text{in } \omega\,. \end{cases} \tag{7}$$

By Proposition 2.1 in [11] (see also [5, Theorem 5]), this problem has a unique solution u_0 and, moreover, $u_0 > 0$ in $\overline{\omega}$. But u_∞ is supersolution for problem (7), so $u_\infty \geq u_0 > 0$ in $\overline{\omega}$ and our claim is proved.

Suppose $\lambda_{\infty,1}$ is finite. Arguing by contradiction, let us assume $a \geq \lambda_{\infty,1}$. Set $\lambda \in (\lambda_1(\mu_0), \lambda_{\infty,1})$ and denote by u_λ the unique positive solution of problem (1). We have

$$\begin{cases} \Delta(Mu_\infty) + \lambda_{\infty,1}(Mu_\infty) \leq b(x)f(Mu_\infty) & \text{in } \Omega\,,\\ Mu_\infty = \infty & \text{on } \partial\Omega\,,\\ Mu_\infty \geq u_\lambda & \text{in } \Omega\,, \end{cases}$$

where $M := \max\{\max_{\overline{\Omega}} u_\lambda / \min_{\overline{\Omega}} u_\infty;\, 1\}$. By the sub-super solutions method we conclude that problem (1) with $\lambda = \lambda_{\infty,1}$ has at least a positive solution (between u_λ and Mu_∞). But this is a contradiction. So, necessarily, $a \in (-\infty, \lambda_{\infty,1})$.

B. Sufficient condition. This will be proved with the aid of several results. From now on we assume throughout the paper that f satisfies (f_1) and (f_2).

Lemma 2. *Let ω be a smooth bounded domain in $\mathbf{R}^N$. Assume p, q, r are $C^{0,\mu}$-functions on $\overline{\omega}$ such that $r \geq 0$ and $p > 0$ in $\overline{\omega}$. Then for any non-negative function $0 \not\equiv \Phi \in C^{0,\mu}(\partial\omega)$ the boundary value problem*

$$\begin{cases} \Delta u + q(x)u = p(x)f(u) - r(x) & \text{in } \omega,\\ u > 0 & \text{in } \omega,\\ u = \Phi & \text{on } \partial\omega, \end{cases} \tag{8}$$

has a unique solution.

Proof. By Lemma 1, problem (8) has at most one solution. The existence of a positive solution will be obtained by device of sub and super-solutions.

Set $m := \inf_\omega p > 0$. Define $\bar{f}(u) = mf(u) - \|q\|_\infty u - \bar{r}$, where $\bar{r} := \sup_\omega r + 1 > 0$. Let t_0 be the unique positive solution of the equation $\bar{f}(u) = 0$. By Remark 1 we derive that $\lim_{u\to\infty} \frac{\bar{f}(u)}{f(u)} = m > 0$. Combining this with (f_2), we conclude that the function $\varphi(w) = \bar{f}(w + t_0)$ defined for $w \geq 0$ satisfies the assumptions of Theorem III in [8]. It follows that there exists a positive large solution for the equation $\Delta w = \varphi(w)$ in ω. Thus the function $\bar{u}(x) = w(x) + t_0$, for all $x \in \omega$, is a positive large solution of the problem

$$\Delta u + \|q\|_\infty u = mf(u) - \bar{r} \quad \text{in } \omega. \tag{9}$$

By Proposition 2.1 in [11], the boundary value problem

$$\begin{cases} \Delta u = \|q\|_\infty u + \|p\|_\infty f(u) & \text{in } \omega, \\ u > 0 & \text{in } \omega, \\ u = \Phi & \text{on } \partial\omega, \end{cases}$$

has a unique classical solution $\underline{u}$. By Lemma 1, we find that $\underline{u} \leq \overline{u}$ in ω and $\underline{u}$ (resp., $\overline{u}$) is a positive sub-solution (resp., super-solution) of problem (8). It follows that (8) has a unique solution. □

Under the assumptions of Lemma 2 we obtain the following result which generalizes Lemma 1.3 in [12].

Corollary 1. *There exists a positive large solution of the problem*

$$\Delta u + q(x)u = p(x)f(u) - r(x) \qquad \text{in } \omega. \tag{10}$$

Proof. Set $\Phi = n$ and let u_n be the unique solution of (8). By Lemma 1, $u_n \leq u_{n+1} \leq \overline{u}$ in ω, where $\overline{u}$ denotes a large solution of (9). Thus $\lim_{n\to\infty} u_n(x) = u_\infty(x)$ exists and is a positive large solution of (10). Furthermore, every positive large solution of (10) dominates u_∞, i.e., the solution u_∞ is the *minimal large solution*. This follows from the definition of u_∞ and Lemma 1. □

Lemma 3. *If $0 \not\equiv \Phi \in C^{0,\mu}(\partial\Omega)$ is a non-negative function and $b > 0$ on $\partial\Omega$, then the boundary value problem*

$$\begin{cases} \Delta u + au = b(x)f(u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = \Phi & \text{on } \partial\Omega, \end{cases} \tag{11}$$

has a solution if and only if $a \in (-\infty, \lambda_{\infty,1})$. Moreover, in this case, the solution is unique.

PROOF. The first part follows with the same arguments as in the proof of Theorem 1.1 (necessary condition).

For the sufficient condition, fix $a < \lambda_{\infty,1}$ and let $\lambda_{\infty,1} > \lambda_* > \max\{a, \lambda_1(\mu_0)\}$. Let u_* be the unique positive solution of (1) with $\lambda = \lambda_*$.

Let Ω_i $(i = 1, 2)$ be subdomains of Ω such that $\Omega_0 \subset\subset \Omega_1 \subset\subset \Omega_2 \subset\subset \Omega$ and $\Omega \setminus \overline{\Omega_1}$ is smooth. We define $u_+ \in C^2(\Omega)$ as a positive function in Ω such that $u_+ \equiv u_\infty$ on $\Omega \setminus \Omega_2$ and $u_+ \equiv u_*$ on Ω_1. Here u_∞ denotes a positive large solution of (10) for $p(x) = b(x)$, $r(x) = 0$, $q(x) = a$ and $\omega = \Omega \setminus \overline{\Omega_1}$. Using Remark 1 and the fact that $b_0 := \inf_{\Omega_2 \setminus \Omega_1} b > 0$, it is easy to check that if $C > 0$ is large enough then $\overline{v}_\Phi = Cu_+$ satisfies

$$\begin{cases} \Delta \overline{v}_\Phi + a\overline{v}_\Phi \le b(x) f(\overline{v}_\Phi) & \text{in } \Omega\,, \\ \overline{v}_\Phi = \infty & \text{on } \partial\Omega\,. \\ \overline{v}_\Phi \ge \max\limits_{\partial\Omega} \Phi & \text{in } \Omega\,. \end{cases}$$

By Proposition 2.1 in [11], there exists a unique classical solution $\underline{v}_\Phi$ of the problem

$$\begin{cases} \Delta \underline{v}_\Phi = |a|\underline{v}_\Phi + \|b\|_\infty f(\underline{v}_\Phi) & \text{in } \Omega, \\ \underline{v}_\Phi > 0 & \text{in } \Omega, \\ \underline{v}_\Phi = \Phi & \text{on } \partial\Omega. \end{cases}$$

It is clear that $\underline{v}_\Phi$ is a positive sub-solution of (11) and $\underline{v}_\Phi \le \max_{\partial\Omega} \Phi \le \overline{v}_\Phi$ in Ω. Therefore, by the sub-super solution method, problem (11) has at least a solution v_Φ between $\underline{v}_\Phi$ and $\overline{v}_\Phi$. Next, the uniqueness of solution to (11) can be obtained by using essentially the same technique as in [4, Theorem 1] or [3, Appendix II]. □

PROOF OF THEOREM 1.1 COMPLETED. Fix $a \in (-\infty, \lambda_{\infty,1})$. Two cases may occur:

CASE 1: $b > 0$ on $\partial\Omega$. Denote by v_n the unique solution of (11) with $\Phi \equiv n$. For $\Phi \equiv 1$, set $v := \underline{v}_\Phi$ and $V := \overline{v}_\Phi$, where $\underline{v}_\Phi$ and $\overline{v}_\Phi$ are defined in the proof of Lemma 3. The sub and super-solutions method combined with the uniqueness of solution of (11) shows that $v \le v_n \le v_{n+1} \le V$ in Ω. Hence $v_\infty(x) := \lim_{n\to\infty} v_n(x)$ exists and is a solution of (2).

CASE 2: $b \ge 0$ on $\partial\Omega$. Let z_n $(n \ge 1)$ be the unique solution of (8) for $p \equiv b + 1/n$, $r \equiv 0$, $q \equiv a$, $\Phi \equiv n$ and $\omega = \Omega$. By Lemma 1, (z_n) is non-decreasing. Moreover, (z_n) is uniformly bounded on every compact subdomain of Ω. Indeed, if $K \subset \Omega$ is an arbitrary compact set, then $d := \operatorname{dist}(K, \partial\Omega) > 0$. Choose $\delta \in (0, d)$ small enough so that $\overline{\Omega}_0 \subset C_\delta$, where $C_\delta = \{x \in \Omega : \operatorname{dist}(x, \partial\Omega) > \delta\}$. Since

$b > 0$ on ∂C_δ, Case 1 allows us to define z_+ as a solution of (2) for $\Omega = C_\delta$. Using Lemma 1 for $p \equiv b + 1/n$, $r \equiv 0$, $q \equiv a$ and $\omega = C_\delta$ we obtain $z_n \leq z_+$ in C_δ, for all $n \geq 1$. So, (z_n) is uniformly bounded on K. By the monotonicity of (z_n), we conclude that $z_n \to \underline{z}$ in $L^\infty_{\rm loc}(\Omega)$. Finally, standard elliptic regularity arguments lead to $z_n \to \underline{z}$ in $C^{2,\alpha}_{\rm loc}(\Omega)$. This completes the proof of Theorem 1.1.

Acknowledgements. We thank the referee for the careful reading of the manuscript and for pointing out that the necessary condition $a < \lambda_{\infty,1}$ in the statement of Theorem 1.1 may be deduced as a consequence of the anti-maximum principle, after showing that the large solution is positive in $\overline{\Omega}_0$.

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Received May 21, 2001

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